Math 441 Name___________________

Quiz 3 Show All Work

Fall 2013

1) Give a linear operator T:V→V, a vector u is called a fixed point if T(**u**) = **u**.

a) Prove that the set of all fixed points of a linear operator T: V→V is a subspace of V.

Let F be the set of all fixed points of the linear operator T. Let **u** and **v** be members of F and let c be a scalar (real number). Clearly F is a subset of V, we must show that F is closed under addition and scalar multiplication. For the first we note that since T is a linear transformation T(**u** + **v**) = T(**u**) + T(**v**) = **u** + **v**. Thus **u** + **v** is a fixed point of T, and F is closed under addition. For the second we note that since T is a linear transformation T(c**u**) = cT(**u**) = c**u.** Thus c**u** is a fixed point of T, and F is closed under scalar multiplication. Q.E.D.

b) Determine all the fixed points of the linear operator $T:\mathbb{R}^2 \to \mathbb{R}^2$ given by T(x,y) =(y,x)

$\{(x, x) \mid x \in \mathbb{R}\}$

2) Find the standard matrix A for the linear transformation T where T is the counterclockwise rotation of 120° in $\mathbb{R}^2$.

$$\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} = \begin{bmatrix} \cos(120°) & -\sin(120°) \\ \sin(120°) & \cos(120°) \end{bmatrix} = \begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$$

3) The linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^3$ is defined by T(**x**) = A**x** where A = $\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 0 & 0 \end{bmatrix}$.

Find a) ker(T); b) nullity(T); c) Range(T); d) Rank(T)

A row reduces to $\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. Thus

a) Ker(T) = $\{(-t, t) \mid t \in \mathbb{R}\}$
b) Nullity(T) = 1
c) Range(T) = $\{t(1, 2, 0) \mid t \in \mathbb{R}\}$
d) Rank(T) = 1

4) Attach the corrected problem from the board (6.3 #58)

Let T: $:\mathbb{R}^2 \to \mathbb{R}^3$ be given by T(x,y) = (x-y, 0, x+y) with B = {(1,2), (1,1)} and B' = {(1,1,1), (1,1,0), (0, 1,1)}.

a) Find T(-3, 2) using the standard matrix
b) Find T(-3, 2) using the matrix relative to B and B'.